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**STRATEGIC RESOURCE EXTRACTION
AND SUBSTITUTE DEVELOPMENT**

By

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Strategic Resource Extraction and Substitute Development

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Abstract

We analyze a dynamic game between a buyer and a seller of an exhaustible resource. The seller chooses resource supply; the buyer can pay a fixed cost to invent a perfect substitute for the resource at any time. In closed-loop equilibrium, the buyer adopts the substitute when the resource is exhausted. Investing makes the buyer worse off because it decreases resource supply, destroys his ability to derive surplus from the resource through delaying the investment cost incurrence, and causes a larger share of the resource stock to be sold at his reservation price. From the seller's perspective, the buyer's ability to develop a substitute is equivalent to an already available substitute with a higher marginal cost.

JEL-Classification: O30, Q30

Keywords: exhaustible resource, substitute, innovation, closed-loop equilibrium

1 Introduction

The oil market has flavours of a bilateral monopoly. The largest exporters have united themselves in OPEC, a cartel that controls more than 75% of proven reserves and actively manages supply. Importing countries coordinate on energy policy and energy security issues through various international organizations such as the IEA, OECD and EU, and cooperate in the development of renewable alternatives. Consuming countries are vulnerable to monopoly power because of

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their heavy dependency on oil, but also have the means to end this dependency through developing a backstop: a substitute that can replace oil as the dominant energy source.

To prolong consumers' dependence on their resource, oil exporters have an incentive to prevent prices from becoming too high. Indeed, one of OPEC's aims is to 'secure an efficient, economic and regular supply of petroleum to consumers'. Conversely, importing countries realize that investment in renewables is not only a remedy to the physical scarcity of oil, but also affects exporters' supply decisions. The resulting strategic interaction is the subject of this paper: we ask how an exhaustible resource seller can adjust the supply path to preserve its monopoly, and how a buyer can optimally use the ability to develop a substitute.

A key feature of exhaustible resource markets is that expectations about future demand affect current supply. A binding promise by the buyer in the initial period about the arrival time of the substitute [Dasgupta et al., 1983, Gallini et al., 1983] will therefore not only change market conditions when the substitute comes on line, but also affect the supply path in all preceding periods [Karp and Newberry, 1993]. As time passes, the effect on supply in early periods becomes sunk, so the buyer faces a different trade-off than in the initial period. As a result, the buyer's optimal open-loop strategy is not time-consistent [Olsen, 1986, 1993]: the buyer has an incentive to commit to late development in order to depress current prices, but would like to renege on his promise when the resource becomes scarce (see section 3). It is unnatural to suppose that the seller is sufficiently rational to calculate dynamic equilibria, but naive enough to believe announcements that are not credible. The contribution of this paper is to derive the closed-loop solution to the investment and supply game.

We use a simple model and a highly stylized representation of the innovation process. In each period, a monopolistic seller makes a supply offer to a buyer. After observing the seller's offer, the buyer decides whether to pay a fixed investment cost and develop a perfect substitute. Upon investment, the substitute can immediately be produced at constant marginal cost and competes with the resource. We abstract from uncertainty, capacity constraints, R&D externalities and imperfect cartelization in order to focus on the strategic aspects of the resource supply and innovation decisions.

In equilibrium, the seller induces the buyer to delay the adoption of the substitute until the resource is exhausted. When the buyer cannot commit to future actions, his only means to influence current supply is to invest immediately. Do-

ing so adversely affects the buyer in three ways. Firstly, the seller immediately reduces supply following investment. Secondly, the buyer loses the ability to derive surplus from the resource through saving the interest on the investment cost. Thirdly, a larger share of the remaining resource stock is sold at the buyer's reservation price. From the seller's perspective, the buyer's ability to develop a substitute is equivalent to an already available substitute with a higher marginal cost. Like in Hoel [1978], the seller limit-prices at the buyer's reservation price, which is higher than the marginal cost of the substitute because of the fixed investment cost. The buyer's indifference condition for investment only becomes binding when the remaining resource stock is sufficiently small.

Our model offers an explanation for the slow progress in renewable energy. Developing a substitute does not help industrialized countries to capture a larger share of the oil rent. So as long as oil prices do not become prohibitively high, there is little economic rationale for substantive efforts.

Four recent papers consider similar questions. Liski and Montero [2011] study the optimal demand schedule of an exhaustible resource monopsonist that has access to a substitute. Their results are similar to ours to the extent that the buyer would like to commit to postpone the switch to the substitute to depress current prices, and the buyer's share of the resource surplus increases in the cost of the substitute when the buyer cannot commit. An important difference is that the perfectly competitive sellers in Liski and Montero [2011] only compete with the substitute for an infinitesimally short period. Because the exhaustion of the resource always coincides with the switch to the substitute, postponing the switch decreases current resource supply. The buyer obtains lower prices because committing to late invention increases his monopsony power. In our framework, the monopolistic seller may compete with the substitute for a nondegenerate period when the buyer can commit the invention time [Olsen, 1993]. Postponing the arrival of the substitute increases cumulative resource supply before invention. Current prices go down because current supply goes up, rather than because of an increase in monopsony power as in Liski and Montero [2011]. As a result, initial resource use is higher under commitment than under discretion in our paper, but lower in Liski and Montero [2011].¹

In Harris and Vickers [1995], the arrival of the substitute is a stochastic event and depends on the buyer's R&D effort. As the buyer's effort increases when the stock dwindles, the seller has an incentive to slow down depletion in closed-loop

¹A formal proof for the general case is beyond the scope of this paper. We provide an example with iso-elastic demand in Appendix D.

equilibrium. Jaakkola [2012] looks at a game with gradual substitute development and convex per-period investment costs. This generalization prohibits an analytical characterization of the closed-loop equilibrium. In Gerlagh and Liski [2011], the substitute is not available immediately following investment, but after an exogenous transition period. By investing, the buyer can force the seller to sell the remaining stock during the transition period. The value of this investment option decreases as the resource is depleted. To compensate the buyer for this decrease, supply is increasing over time during an interval before investment. Although Gerlagh and Liski look for a Markov-perfect equilibrium, the time-to-build delay acts as a commitment device.

2 Model

Consider a model with a buyer and a seller of an exhaustible resource. We adopt the notation of Gerlagh and Liski [2011]. The buyer derives flow surplus $u(q_t)$ from consuming q_t units of the resource. Let $u(\cdot)$ be increasing and continuously differentiable. The surplus function is associated with a consumer utility function $\tilde{u}(\cdot)$ satisfying $u(q_t) = \tilde{u}(q_t) - \tilde{u}'(q_t) q_t$ and inverse demand function $\psi(q_t) = \tilde{u}'(q_t)$. A monopolistic seller is endowed with a finite stock s_0 that is costless to extract. The seller aims to maximize the discounted stream of instantaneous profits $\pi(q_t) = \tilde{u}'(q_t) q_t$. Utility and profits are discounted at rate r .

The buyer can instantaneously develop a perfect substitute for the resource. By paying an upfront investment cost I , he gains the ability to produce the substitute at constant marginal cost c . The buyer is then guaranteed a flow surplus

$$\bar{u} \equiv u(\psi^{-1}(c))$$

There are $i = 1, \dots, N$ periods of time of length ε . Time is continuous but agents only act at the beginning of each period, i.e. at time points $t_i = \varepsilon(i - 1)$. They commit their actions for the entire period. We distinguish between a pre-investment phase A and a post-investment phase B . In phase A , the buyer has a binary decision variable $k_{t_i} \in \{0, 1\}$, where $k_{t_i} = 1$ indicates that the buyer develops the substitute. In phase A , each period has three stages:

- (1) the seller offers to supply a quantity q_{t_i}
- (2) the buyer chooses $k_{t_i} \in \{0, 1\}$
- (3) when $k_{t_i} = 0$, the market clears at price $p_{t_i} = \psi(q_{t_i})$, or when $k_{t_i} = 1$, the seller's offer is rejected and the economy moves to regime B

In phase B , the seller is the only mover and chooses quantity q_{t_i} . The market clears at price $p_{t_i} = \min(\psi(q_{t_i}), c)$, because the buyer cannot prevent the seller from competing with the substitute.

Our timing reflects the mutual dependence of buyers and sellers in natural resource markets, and oil exporters' concerns about security of supply: if the supply offer is sufficiently generous, the buyer will refrain from investing. Olsen [1993] uses the same setup as our paper, but the buyer moves before the seller in each period. This timing forces the buyer to invest very early, as he cannot credibly punish the seller for setting a high price.²

3 The time-consistency problem

Olsen [1993] sketches why equilibria in which the buyer can commit the investment time are not consistent. According to the Hotelling rule, a monopolistic seller equates discounted marginal revenue in all periods. When the arrival time of the substitute T' is fixed, additional supply in any period after T' does not cause inframarginal losses because the resource is sold at the substitute price. As marginal revenue is relatively high after the arrival of the substitute, the seller would like to sell part of his stock in the post-investment phase. This is socially inefficient and reduces the buyer's welfare, because the seller directly displaces substitute supply using scarce resource units. When the buyer commits to a late T' , the seller's post-investment revenues are discounted at a higher rate. This makes it less attractive for the seller to compete with the substitute, thereby reducing cumulative post-investment supply.

A late T' results in higher cumulative extraction in the pre-investment phase and increased supply in early periods, but is costly for the buyer in later periods: when the resource becomes scarce, the buyer cannot dispose over the substitute as early as he would like. After having enjoyed the benefits of high supplies and low prices early on, the buyer has an incentive to avoid the costs of resource scarcity in subsequent periods by developing the substitute earlier than announced. We illustrate this incentive in Figure 1. After \hat{T} , supply drops below the buyer's reservation quantity \check{q} , which is defined such that $u(\check{q})$ equals

²Olsen does not analyze large initial stocks and restricts himself to the limit-pricing phase. For large stocks, the buyer cannot credibly prevent the seller from treating his problem as a free-terminal-time problem with a scrap value, the scrap value being the discounted profits from the limit-pricing phase. The buyer thus invests before the remaining stock reaches the region that Olsen considers.

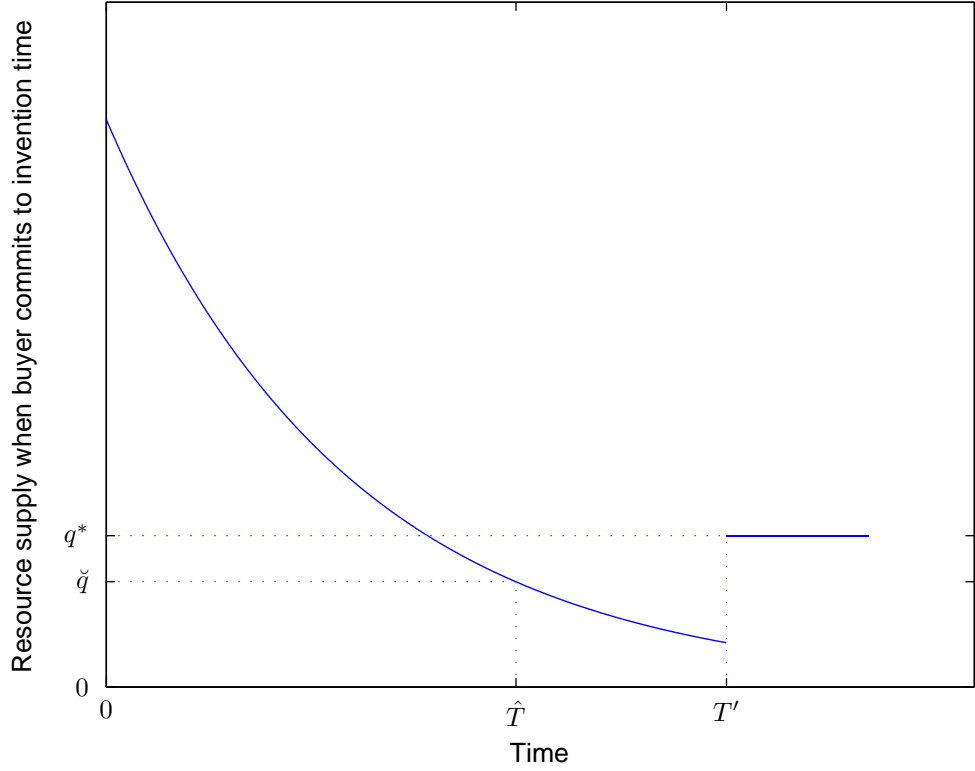


Figure 1: The buyer's time-consistency problem

the utility $u(q^*)$ of consuming the long-run substitute supply q^* minus the interest on the investment cost rI . After \hat{T} , the buyer prefers to invest immediately instead of at the announced time T' .

4 Post-investment phase

When making decisions during phase A , agents take into account their payoffs in phase B . We therefore first turn to phase B . When the period length ε is sufficiently small, we can approximate the seller's problem in phase B by its

continuous-time analogue:

$$\begin{aligned} & \max_{q_t} \int_0^\infty \min(\psi(q_t), c) q_t e^{-rt} dt \\ & \text{s.t. } \dot{s} = -q_t, \quad q_t \geq 0, \quad s_t \geq 0, \quad s_0 \text{ given} \end{aligned} \quad (1)$$

The Hamiltonian for this problem and the necessary optimality conditions are

$$\begin{aligned} H(q, s, \lambda) &= \min(\psi(q_t), c) q_t - \lambda \dot{q} \\ \pi'(q_t) - \lambda_t &\leq 0, \quad q \geq \psi^{-1}(c) \quad \text{c.s.} \\ \dot{\lambda}_t &= r\lambda_t, \quad \lim_{t \rightarrow \infty} \lambda_t s_t = 0 \end{aligned} \quad (2)$$

where λ is the scarcity value of the resource and

$$\pi'(q_t) = \begin{cases} \psi(q_t) - q_t \psi'(q_t) & \text{for } q_t \geq D(c) \\ c & \text{for } q_t < D(c) \end{cases}$$

Because a perfect substitute is available at cost c , marginal revenue $\pi'(q_t)$ is equal to c for the first $q^* \equiv \psi^{-1}(c)$ units and discontinuous at q^* , as the seller starts incurring losses on inframarginal units. If the seller practices a limit-pricing strategy, supplying q^* in each period until exhaustion, the stock is exhausted at time s/q^* . For small values of the remaining stock, the marginal revenue $\pi'(q_t)|_{q_t \geq q^*}$ of supplying a unit in excess of q^* is always lower than the present value of c at time s/q^* . The seller then supplies q^* in each period until exhaustion time T . When the remaining stock is large, limit pricing takes so long to exhaust the stock that $\pi'(q^*)$ is higher than ce^{-rs_t/q^*} . It is then worthwhile to supply $q_t > q^*$ in early periods.

Proposition 1 (post-investment phase, Hoel [1978]). *Let*

$$\begin{aligned} s^* &\equiv -\frac{q^*}{r} \ln \left(\frac{\pi'(q^*)}{c} \right) \\ \Delta &\equiv \frac{s^*}{q^*} \end{aligned}$$

When the remaining stock $s < s^$*

$$q_t = q^* \quad \forall t \in (0, T), \quad T = \frac{s}{q^*}$$

When $s > s^$*

$$\begin{aligned} q_t &= \pi'^{-1} \left(ce^{-r(T-t)} \right) \quad \forall t \in [0, T - \Delta) \\ q_t &= q^* \quad \forall t \in [T - \Delta, T] \end{aligned} \quad (3)$$

$$\text{s.t. } \int_0^{T-\Delta} \pi'^{-1} \left(ce^{-r(T-t)} \right) dt + \Delta q^* = s \quad (4)$$

The above system defines the seller's optimal post-investment strategy $\phi^I(s; c)$.

Figure 2 illustrates the seller's supply path during the post-investment phase. In equilibrium, the seller is indifferent between supplying an extra unit in any period before $T - \Delta$ and marginally extending the limit-pricing period. The present value of marginal revenue $\pi'(q_t)e^{-rt}$ equals ce^{-rT} between $[0, T - \Delta]$ and at T .

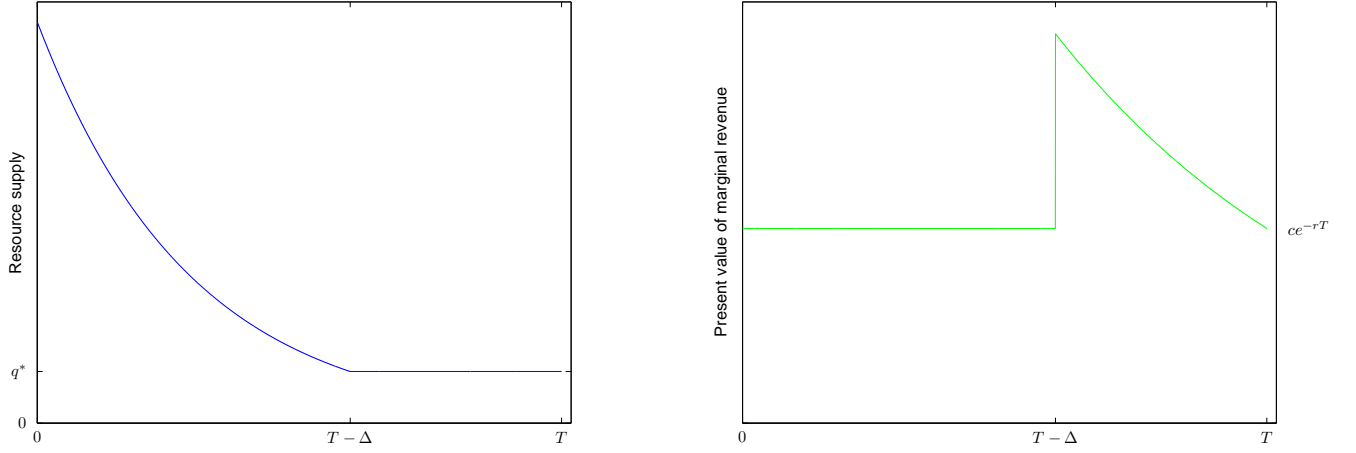


Figure 2: The seller's optimal supply (left) and the present value of marginal revenue (right) during the post-investment phase

5 Closed-loop equilibrium

It will be useful to define the value function for the seller $V(s)$ and buyer $U(s)$ as a function of the remaining stock during phase A. Denote the buyer's and seller's value at the time of investment by $U^I(s)$ and $V^I(s)$, respectively. Let $\kappa(s, q)$ be the buyer's investment strategy and $\phi(s)$ the seller's supply strategy. The value for the seller is the payoff from choosing the optimal q for an interval of length ε , given the strategic response of the buyer

$$V(s) = \max_{\{q\}} \{[\varepsilon\pi(q) + e^{-\varepsilon r}V(s - \varepsilon q)](1 - \kappa(s, q)) + V^I(s)\kappa(s, q)\}. \quad (5)$$

We define the value for the buyer analogously

$$U(s) = \max_{k \in \{0,1\}} \{[\varepsilon u(\phi(s)) + e^{-\varepsilon r}U(s - \varepsilon\phi(s))](1 - k) + U^I(s)k\}. \quad (6)$$

The value of the resource to the buyer $W(s)$ is the utility it provides on top of the long-run level

$$W(s) = U(s) - \left(\frac{\bar{u}}{r} - I \right)$$

The post-investment values $U^I(s)$ and $V^I(s)$ are determined by the dynamics described in Proposition 1.

$$W^I(s) = \int_0^{T-\Delta} [u(q_t) - \bar{u}] e^{-rt} dt \quad (7)$$

We look for a pair of equilibrium strategies such that $\phi(s) = \operatorname{argmax}_{\{q\}} V(s)$ and $\kappa(s, \phi(s)) = \operatorname{argmax}_k U(s)$ for all s . Because the buyer can guarantee himself the long-run surplus $\bar{u} - rI$ by investing, the buyer's strategy maximizes his value of the resource $W(s)$. Proposition 2 states the main result.

Proposition 2. *There exists a subgame-perfect equilibrium in pure strategies. The buyer's equilibrium strategy is*

$$\kappa(s, q) = 1 \quad \forall q : u(q) < \bar{u} - rI \quad (8a)$$

$$\kappa(s, q) = 0 \quad o.w. \quad (8b)$$

The seller's equilibrium strategy is $\phi(s) = \phi^I(s, \psi(u^{-1}(\bar{u} - rI)))$, with $\phi^I(s; c)$ as defined in Proposition 1.

The buyer's reservation utility in the post-investment phase is $u(q^*) = \bar{u}$, the utility he attains by consuming the substitute. The seller always supplies $q \geq q^*$, because there is no loss on inframarginal units for smaller q . The buyer's reservation utility before investment is $u(\check{q}) = \bar{u} - rI$, reflecting that resource consumption allows him to delay investment. The seller's best response to the buyer's equilibrium strategy is to supply at least $q \geq \check{q}$, because he would trigger investment for smaller q .

Although the motivation differs in the pre- and post-investment phase, the seller always supplies at least the buyer's reservation quantity. From the seller's perspective, the threat of the substitute is equivalent to an already available substitute with a higher marginal cost. Adopting the substitute does not change the equilibrium dynamics except for raising the buyer's reservation utility from $\bar{u} - rI$ to \bar{u} . This is not accompanied by an increase in actual utility, as the buyer pays interest rI on the sunk investment cost. The increase in the buyer's reservation utility has three effects, as we show in Figure 3. Firstly, the buyer's surplus from the resource ($u(q)$ minus the reservation utility) is lower for any given supply path: $u(q) - u(\check{q}) > u(q) - u(q^*)$. Secondly, the seller decreases

supply in early periods when the buyer invests: $\phi(s) > \phi^I(s)$ when $s > s^*$ [Hoel, 1978, 1983]. The maximum price for the resource decreases after investment; to compensate for this, the seller increases the price in early periods. Thirdly, a larger share of the resource stock is sold at the buyer's reservation price, that is, $\check{q}(\check{T} - \check{\Delta}) < q^*(T - \Delta)$. Combining these effects, the buyer's surplus from the resource decreases after investment.

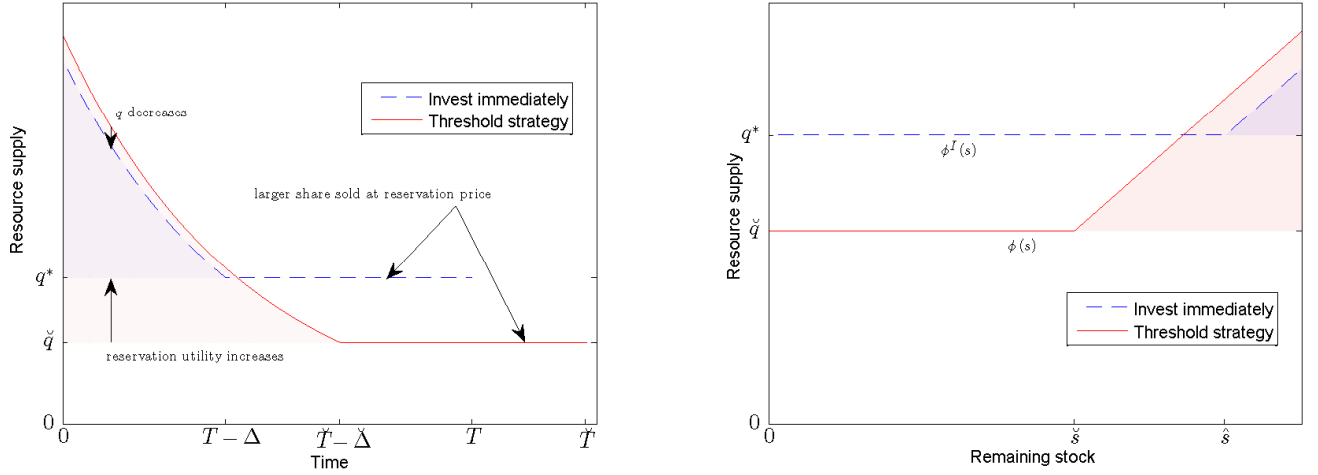


Figure 3: Resource supply before and after investment as a function of time (left) and the remaining stock (right)

Corollary 1. $W(s) \downarrow 0$ when $r \downarrow 0$

The buyer receives a positive surplus from the resource when the seller is too impatient to sell the entire stock at the buyer's reservation price. As r approaches zero, this consideration vanishes and so does $W(s)$.

In the Appendix, we derive the same results when the buyer can invest a continuous amount in each period and the marginal cost of the substitute depends on cumulative investment. The buyer invests the amount that maximizes consumption utility minus the interest on the investment costs in one go when the resource is exhausted. Similarly, the seller supplies at least the buyer's long-run utility until exhaustion.

6 Conclusion

Strategic exhaustible resource buyers have an incentive to delay the adoption of a substitute in order to depress current prices. When the buyer cannot commit to the arrival time of the substitute, a monopolistic seller can induce the buyer to delay adoption until exhaustion. In the absence of uncertainty or a fixed time-to-build delay, the threat of a substitute has a similar effect as an already available substitute in closed-loop equilibrium. The model is highly stylized - for example, OPEC and industrialized economies both face substantial coordination and free-riding problems that impede their ability to act as a monopolist or jointly develop a substitute. Nonetheless, the results accord with the slow technological progress in renewable energy and oil exporters' efforts to stabilize prices.

A Proof of Proposition 1

The seller supplies q^* at time t when the MR of supplying an additional unit is lower than the discounted MR at the time of exhaustion, i.e.

$$\pi'(q^*) \leq \lambda_t = \lambda_T e^{-r(T-t)} = ce^{-r(T-t)}$$

Solving for $T - t$ yields

$$T - t \leq -\frac{1}{r} \ln \left(\frac{\pi'(q^*)}{c} \right) \quad (9)$$

The seller supplies q^* in each period until exhaustion when this inequality holds for all $t \in \left(0, \frac{s}{q^*}\right)$. We can calculate the threshold stock s^* such that the seller practices limit pricing until exhaustion for all $s \leq s^*$ by substituting $T = \frac{s}{q^*}$ and solving for the s such that (9) holds with equality for $t = 0$

$$s^* = -\frac{q^*}{r} \ln \left(\frac{\pi'(q^*)}{c} \right)$$

When $s > s^*$, the seller extracts the last s^* units in a period of length $\Delta = \frac{s^*}{q^*}$. While extracting the first $s - s^*$ units, marginal revenue rises at the rate of interest and is equal the present value of marginal revenue at the time of exhaustion

$$\begin{aligned} \pi'(q_t) &= ce^{-r(T-t)} \\ \Rightarrow q_t &= \pi'^{-1} \left(ce^{-r(T-t)} \right) \end{aligned}$$

Let $\eta(\cdot) = (\pi')^{-1}$ be the seller's supply as a function of marginal revenue. The stock constraint determines T

$$\int_0^{T-\Delta} \eta\left(ce^{-r(T-t)}\right) dt + \Delta q^* = s \quad (10)$$

B Proof of Proposition 2

We make use of a special case of Proposition 3 in Hoel [1983].

Lemma 1 (post-investment phase, Hoel [1983]). *When $s > s^*$, $\phi^I(s; c)$ is increasing in c .*

Because of the buyer's investment option, the resource is exhausted in finite time. This means there exists a subgame-perfect equilibrium in pure strategies. We assume that the number of periods N is sufficiently large, such that the optimal strategies are not affected by the length of the game. The buyer's strategy maximizes $U(s)$ with respect to k . His value from investing is

$$\begin{aligned} U^I(s) &= \int_{T-\Delta}^{\infty} \bar{u} e^{-rt} dt - I + \int_0^{T-\Delta} u(q_t) e^{-rt} dt \\ &= \int_0^{\infty} (\bar{u} - rI) e^{-rt} dt + \int_0^T [u(q_t) - \bar{u}] e^{-rt} dt, \quad q_t \text{ s.t. } u(q_t) \geq \bar{u} \quad \forall t \in (0, T) \\ &= \int_0^{\infty} (\bar{u} - rI) e^{-rt} dt + W^I(s; c) \end{aligned}$$

Let $\check{U}(s)$ denote the payoff when the buyer adopts the equilibrium strategy, and denote the corresponding exhaustion time by \check{T} . We have

$$\begin{aligned} \check{U}(s) &= \int_{\check{T}}^{\infty} \bar{u} e^{-rt} dt - e^{-r\check{T}} I + \int_0^{\check{T}} u(q_t) e^{-rt} dt \\ &= \int_{\check{T}}^{\infty} (\bar{u} - rI) e^{-rt} dt + \int_0^{\check{T}} u(q_t) e^{-rt} dt \\ &= \int_0^{\infty} (\bar{u} - rI) e^{-rt} dt + \int_0^{\check{T}} (u(q_t) - [\bar{u} - rI]) e^{-rt} dt, \quad q_t \text{ s.t. } u(q_t) \geq \bar{u} - rI \quad \forall t \in (0, \check{T}) \end{aligned}$$

Then there exists a $\check{c} \geq c$ such that

$$\check{U}(s) = \int_0^{\infty} (\bar{u} - rI) e^{-rt} dt + W^I(s; \check{c})$$

In order to prove that $\check{U}(s) \geq U^I(s)$, it is sufficient to show that $W^I(s; c)$ is weakly increasing in c . By Lemma 1, $\phi^I(s; c)$ increases in c for $s > s^*$.

Moreover, \bar{u} decreases in c . Then $T - \Delta$ increases in c . Therefore, $W^I(s; c)$ is strictly increasing in c when $s > s^*$, that is, when the limit-pricing phase has not yet started. When $s \leq s^*$, observe that $\frac{\partial q^*}{\partial c} < 0$ and $\frac{\partial \Delta}{\partial c} \leq 0$, so $\frac{\partial s^*}{\partial c} \leq 0$. When the seller is limit pricing, he will keep limit-pricing after a reduction in c . Thus, $\frac{\partial W^I(s; c)}{\partial c} = 0$ for $s \leq s^*$.

The seller always supplies at least \check{q} given that the buyer only invests when $q < \check{q}$. When the seller chooses a $q_t < \check{q}$, he triggers immediate investment and is subject to the more stringent constraint $q_t \geq q^*$ for the remainder of the game. Choosing $q_t < \check{q}$ therefore cannot be optimal, provided the period length ε is sufficiently small.

C Continuous Investment

In this section, we generalize the buyer's problem: rather than making a discrete investment decision, the buyer can invest a continuous amount in each period. The marginal cost of the substitute is a decreasing function of cumulative investment

$$c = c(I), \quad c'(I) \leq 0, \quad I = \int_0^\infty idt$$

We can write $\bar{u}(I) \equiv u(\psi^{-1}(c(I)))$. The timing in each stage is

- (1) the seller supplies a quantity q_{t_i}
- (2) the buyer chooses i
- (3) the market clears at price $p_{t_i} = \min(\psi(q_{t_i}), c(I))$

Profits and utility depend on the stance of technology I in addition to the supply q . Let $\phi(s, I)$ denote the seller's supply strategy and $\iota(s, I, q)$ the buyer's investment strategy. The value functions are given by

$$V(s, I) = \max_{\{q\}} \{ \varepsilon \pi(I, q) + e^{-\varepsilon r} V(s - \varepsilon q, I + \varepsilon \iota(s, I, q)) \} \quad (11)$$

$$U(s, I) = \max_{\{i\}} \{ \varepsilon (u(I, \phi(s, I)) - i) + e^{-\varepsilon r} U(s - \varepsilon \phi(s, I), I + \varepsilon i) \} \quad (12)$$

Analogously to the main text, we can express the value of the resource to the buyer as

$$W(s, I) = U(s, I) - \left(\frac{\bar{u}(I)}{r} - I \right)$$

Utility from the substitute is a concave function of cumulative investment.

Assumption 1. $\frac{\partial \bar{u}}{\partial I} \geq 0$, $\frac{\partial^2 \bar{u}}{\partial I} < 0$

The equilibrium is similar to the one in Proposition 2. The buyer selects the cumulative investment level I^* that yields the highest long-run utility $\bar{u} - rI$, and invest this amount in one go when the resource is exhausted.

Proposition 3. *Let I^* be the solution to $\frac{\partial \bar{u}}{\partial I} = r$. There exists a subgame-perfect equilibrium in pure strategies. The buyer's equilibrium strategy is*

$$\iota(s, 0, q) = I^* \quad \forall q : u(q) < \bar{u}(c(I^*)) - rI^* \quad (13a)$$

$$\iota(s, I, q) = 0 \quad o.w. \quad (13b)$$

and the seller's equilibrium strategy is

$$\phi(s, I) = \phi^I(s; \min(c(I), u^{-1}(\bar{u}(c(I^*)) - rI^*))) \quad (14)$$

with $\phi^I(s; c)$ as defined in Proposition 1.

Proof. First, we show that given the seller's strategy (14), the buyer's strategy must satisfy $\vec{I} \equiv \int_0^\infty idt = I^*$. Suppose that for a certain buyer's strategy $\vec{I} < I^*$. Then by Assumption 1, the buyer can marginally improve his welfare by choosing $\iota(0, \vec{I}, 0) > 0$. Suppose that $\vec{I} > I^*$. Then the buyer must either invest a positive amount when cumulative investment already exceeds I^* , or invest a large amount when $I < I^*$ such that cumulative investment overshoots I^* . We discuss both cases in turn. Firstly, suppose there exists an I such that $I^* \leq I$ and $\iota(s, I, \phi(s, I)) > 0$ for some s . Let I' denote the largest such I . Then the buyer can marginally improve his welfare by choosing $\iota(s, I', \phi(s, I')) = 0$. By Assumption 1, this increases the long-run utility $\bar{u} - rI$, and by Proposition 2, it increases the value of the resource $W(s, I)$. Secondly, suppose there exists an $I < I^*$ such that $\iota(s, I, \phi(s, I)) > I^* - I$ for some s . Let I'' be the largest such I . Then the buyer can marginally improve his welfare by choosing $\iota(s, I'', \phi(s, I'')) = I^* - I''$. Again, this improves long-run utility by Assumption 1 and increases the value of the resource by Proposition 2. By induction it follows that cumulative investment \vec{I} must equal I^* .

Secondly, we show that given the seller's strategy (14), it is optimal to invest I^* in one go when the resource is exhausted. When the resource is exhausted, it is optimal for the buyer to reach the long-run optimal level I^* as quickly as possible. Suppose that the buyer's strategy entails $i_1 = \iota(0, I_1, 0) > 0$, $i_2 = \iota(0, I_2, 0) > 0$ for some $0 < I_1 < I_2 < I^*$. Then by Assumption 1, the buyer can marginally improve his welfare by choosing $\iota(0, I_1, 0) = I_2 - I_1 + i_2$. By induction it follows that the buyer invests $I^* - I$ when the resource is exhausted.

When the resource is not yet exhausted, it is optimal to delay investments until exhaustion. Suppose there exists an $s > 0$ such that $\iota(s, I, \phi(s, I)) > 0$ for some I . Let s_3 be the lowest such s . Then the buyer can marginally improve his welfare by choosing $\iota(s_3, I, \phi(s_3, I)) = 0$ and increasing the investment at exhaustion by the same amount. This does not affect the long-run utility $\bar{u}(c(I^*))$, but increases $W(s, I)$ by Proposition 2.

Thirdly, the proof that the seller's strategy (14) is optimal given buyer's strategy (13) is analogous to the last part of the proof of Proposition 2. \square

D Commitment and subgame perfect equilibria

In this section, we provide an example with an iso-elastic demand function in which a monopsonistic buyer that already has developed a substitute, as in Liski and Montero [2011], chooses a lower initial level of resource consumption under commitment than in closed-loop equilibrium. In the bilateral monopoly framework that we study in this paper, initial consumption is higher when the buyer can commit the invention time than in closed-loop equilibrium.

Let $u(q) = \sqrt{q}$, which corresponds to an iso-elastic demand with elasticity 2. We numerically compute the resource consumption paths in Liski and Montero's framework when the buyer can fully commit q_t for all t , in closed-loop equilibrium (where the buyer's strategy $q_t = C(S_t)$ is a function of the remaining stock) and in the social optimum. When the buyer has full commitment power, his problem is

$$\begin{aligned} V_{t=0} &= \max_{\{q_t, T\}} \int_0^T \{\sqrt{q_t} - p_t q_t\} e^{-rt} dt + \frac{1}{r} \frac{1}{4c} e^{-rT} \\ \text{s.t. } \dot{S}_t &= -q_t, S_0 > 0, S_T = 0 \\ \dot{p}_t &= r p_t, p_T = c \end{aligned}$$

where T denotes the time at which the resource is exhausted and the buyer switches to the substitute. Liski and Montero, pp. 6 show that the buyer's commitment optimum is fully characterized by

$$\frac{1}{2\sqrt{q}} + \frac{1}{4r} q^{-1\frac{1}{2}} \frac{\partial q}{\partial t} = 0 \quad (15a)$$

$$\frac{1}{4c} - \frac{1}{2} \sqrt{q_T} = c r s_0 \quad (15b)$$

In closed-loop equilibrium, the buyer's value function is

$$V(S_t) = \int_t^\infty \left[\sqrt{C(S_\tau)} - P(S_\tau) C(S_\tau) \right] e^{-r(\tau-t)} d\tau$$

where $q_t = C(S_t)$ and $p_t = P(S_t)$ are the equilibrium consumption and pricing rule, respectively. The authors demonstrate (pp. 9) that the equilibrium is given by a first-order ODE in the pricing rule

$$\frac{1}{2} \sqrt{-\frac{P'(S)}{rP(S)}} - P(S) + P'(S)S = 0 \quad (16)$$

with boundary condition $P(0) = c$. Lastly, the social optimum is characterized by the Hotelling rule, $p_t = u'(q_t)$ and $p_T = c$ and has an analytical solution: $T = \frac{1}{2r} \ln(1 + 8rs_0c^2)$. Using MATLAB, we numerically solve (15) and (16) to determine the consumption paths under commitment and in closed-loop equilibrium for $s_0 = 20, c = 0.5, r = 0.04$.³ Figure 4 depicts the results. In line with the intuition in the main text, resource use in Liski and Montero [2011] is lower under commitment than in closed-loop equilibrium. When the buyer has full commitment power, $p_t = p_T e^{-r(T-t)}$. By committing to a late switch to the substitute, the buyer reduces initial extraction but obtains a substantial price discount.

Now return to the bilateral monopoly model in this paper. Dasgupta et al. [1983] and Olsen [1986] discuss the buyer's optimal invention time under commitment at length. We demonstrate the following result through a series of Lemmas.

Proposition 4. *For iso-elastic demand with elasticity equal or larger than 2 and a sufficiently large initial stock, initial resource use is higher when the buyer commits the invention time at $t = 0$ than in closed-loop equilibrium.*

Lemma 2 (Proposition 1, Olsen [1986]). *For iso-elastic demand with elasticity equal or greater than 2 and s_0 sufficiently large, the seller has zero stock remaining at the buyer's optimal invention time.*

Lemma 3 (Dasgupta et al. [1983], pp. 1442). *Resource supply at $t = 0$ increases in the invention time between 0 and the buyer's optimal invention time.*

Lemma 4 (Dasgupta et al. [1983], pp. 1442). *For iso-elastic demand with elasticity equal or greater than 2 and s_0 sufficiently large, resource supply at $t = 0$ decreases in the buyer's invention time when the invention time is greater than the buyer's optimum.*

Lemma 1 implies that resource supply at $t = 0$ when the buyer commits to an infinitely large invention time (which is equivalent to $c \rightarrow \infty$) is larger than initial resource supply in closed-loop equilibrium. Then Lemmas 2, 3 and 4 establish the Proposition.

³The code is available upon request.

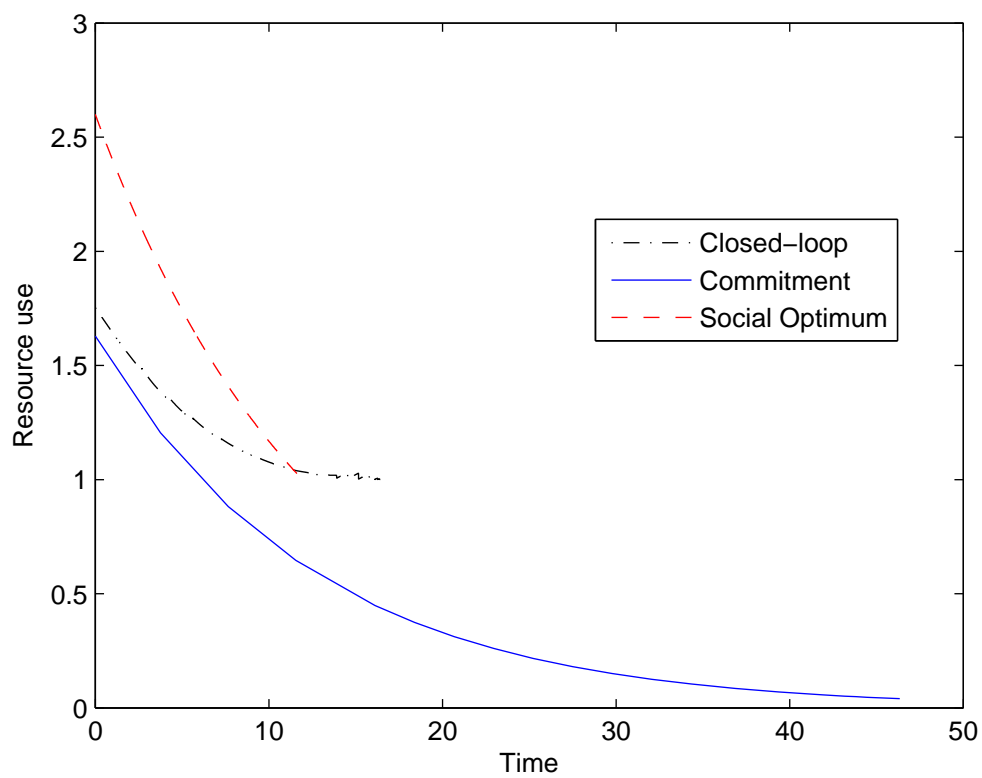


Figure 4: Resource consumption in Liski and Montero [2011]

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